Face recognition

End-semester project
Vector and Matrices 2

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May, 2022

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Aim of the Project

We start with a bunch of facial photographs, 5 photos of 10 persons each. All the images are $200 * 180$ in size. So they are 36000 dimensional vectors. Our aim is to reduce the dimension as much as possible without losing our ability to recognise the faces and after that, we want the machine to understand the difference between photos of different persons and to find the similarity of the photos of same person.

Some Mathematical Results

If $X_1, X_2, ...X_n$ are n points of which variance or separation we want to see, then we know that if $Y_i's$ are the projections of $X_i's$ then $Y_i = u^T X_i$ implies $Var(Y_1, Y_2, ...Y_n) = u^T Var(X_1, X_2, ..., X_n)u$, WLOG, let u is an unit vector. Hence, $Max(Var(Y_1, ...Y_n)) = max_{\mathrm{u} \neq 0}((u^T Var(X_1, X_2, ..., X_n)u))$ over unit vectors. Now if we want to maximize the variance between different classes and minimize between same classes, then we also have to take in account, the inter-class variance. So, we have to now maximize $u^T Bu$ and minimize $u^T Wu$, where B is the variance covariance matrix of the whole data each cluster replaced by its mean and W is the sum of the variance covariance matrices between the clusters.

Some Mathematical Results

So, we have to maximize $\frac{u^T B u}{u^T W u}$ when u is not null and W is P.D. and both are square matrices.

Now $W = M^T M$ [as W is P.D., hence M is n.s.]
now let $Mu = y, u = M^{-1} y$ [as M is n.s.]
Hence, $max_{y \neq 0} \frac{y^T (M^{-1})^{T} B M^{-1} y}{y^T y}$ is our desired result. Now we know that by spectral decomposition if A is real symmetric matrix, then $max_{y \neq 0} \frac{y^T A y}{y^T y}$ is the largest eigen-value of A.Now, here A is $(M^{-1})^{T} B M^{-1}$ Let us check whether it is real symmetric or not. First of all, we are in real field and $A^T = ((M^{-1})^{T} B^T ((M^{-1})^{T})^{T}) = (M^{-1})^{T} B M^{-1} = A$[as B is symmetric]

Some Mathematical Results

Hence, the maximum value of the expression will be the largest eigen value of $(M^{-1})^T BM^{-1}$. Now, all are square matrices. And we know that eigen value of AB and BA are same if A,B are square matrices. Hence, eigenvalue of $(M^{-1})^T BM^{-1}$=eigen value of $M^{-1}(M^{-1})^T B$=eigen value of $W^{-1}B$
Now $(M^{-1})^T BM^{-1}Mu_0 = \lambda Mu_0$
Hence, $W^{-1}Bu_0 = \lambda u_0$
Hence, u_0 will be the eigen vector corresponding to the largest eigen value of $W^{-1}B$.

Total work

Hence, by first applying PCA, we reduced our data from $50 * 36000$ to $50 * 10$. Now we will apply the above mentioned procedure to get our desired result. Hence we will get 10 different clusters each containing 5 different points.Then we will separate those by checking the range and can almost surely distinguish between different or same set of photos by machine.